ENCYCLOPEDIA on Early Childhood Development

NUMERACY

Early Predictors of Mathematics Achievement and Mathematics Learning Difficulties

[1]Nancy C. Jordan, PhD, [2]Brianna L. Devlin, PhD

[1]University of Delaware, USA, [2]University of Oregon, USA

December 2023, Éd. rév.

Introduction

Mathematics difficulties are widespread. Up to 10% of students are diagnosed with a learning disability in mathematics at some point in their school careers.[1,2] Many more learners struggle in mathematics without a formal diagnosis. Mathematics difficulties are persistent, and students who have difficulties may never catch up to their normally achieving peers without intervention.

Subject

Foundations for mathematics achievement are established before children enter primary school.[3,4] Identification of key predictors of mathematics outcomes provides support for screening, intervention and progress monitoring before children fall seriously behind in school.

Problem

The consequences of poor mathematics achievement are serious for everyday functioning, educational attainment, and career advancement.[5] Mathematics competence is necessary for entry into STEM (science, technology, engineering and mathematics) disciplines in college and for STEM-related occupations.[6] There are large group differences in mathematics achievement related to socioeconomic status[7] as well as individual differences in general learning abilities.[8] These disparities are already present in early childhood and increase over the course of schooling.

Research Context

Longitudinal studies of characteristics of children with mathematics difficulties have identified important targets for intervention. Most children enter school with *number sense* that is relevant to learning school mathematics. Preverbal components of number (e.g., perceiving exact representations of small sets of objects and approximate representations of larger sets) develop in infancy.[9,10,11,12] Although these primary foundations are thought to underlie learning of conventional mathematics skills, they are not sufficient. Most children with difficulties in mathematics show weaknesses in number sense related to knowledge of number, number relations, and number operations[4,13] – strands of number sense are malleable and influenced by experience.[14] Early number relates to knowledge of oral and written number and counting concepts, such as one-to-one correspondence and cardinality. Number relations involve understanding of numerical magnitudes on the number line. Number operations relates to transforming quantities through addition and subtraction.[4,15]

Key Research Questions

Early competencies that are aligned with the mathematics children are required to do in school are most predictive of mathematics achievement and difficulties.[16] Assessment tools and interventions need to be refined to help children develop key number sense concepts. Identifying pathways and factors related to number sense development is needed to guide the creation of early childhood interventions for those at-risk for developing mathematics learning difficulties in school.

Recent Research Results

Early number sense sets children's achievement trajectories in mathematics.[16,17,18,19,20] Mathematics difficulties and disabilities have their roots in poorly developed number sense.[21,22,23] Children with developmental dyscalculia, a severe form of mathematics disability, are characterized by deficits in counting and enumerating sets of objects and in recognizing and comparing numbers.[21] Such deficits lead to poor arithmetic fluency, a hallmark skill in the primary grades.

Number sense as a predictor of later mathematics achievement and difficulty

Studies provide empirical evidence of a multifactor early number sense model consisting of specific strands of number, number relations, and number operations understanding.[24,25] Early screening tools developed using this model accurately identify children at risk for mathematics learning difficulty and disability.[26,27,28] Predictive relations may differ by level of number sense in preschool. Number strand skills predict later mathematics achievement for children with low and intermediate achievement, but not high achievement; number relations strand skills predict later achievement for children of all math achievement levels; number operations strand skills predict later math achievement for children with intermediate and high achievement, but not low achievement.[29]

Low-income children enter kindergarten well behind their middle-income peers on most symbolic numeracy indicators, and this gap does not narrow during the school year.[13] Longitudinal studies over multiple time points, from the beginning of kindergarten through the end of Grade 3, suggest that foundational number sense supports the learning of complex mathematics associated with computation as well as applied problem solving and fluency.[16,20,30,31] The low mathematics achievement of high-risk, low-income students is mediated by kindergarten number sense. Because early number competencies are achievable in most children[4] their intermediate effects provide clear directions for early intervention

Type of quantity representation and set size

The size of a quantity or set along with the way it is presented to a child (e.g., non-symbolic or symbolic) affects children's reasoning about numbers. Children's ability to map written numerals to the quantities they represent is critical for learning more complex number sense skills.[32] Non-symbolic quantity representations (e.g., which of 2 sets of dots has more, without counting) scaffold the development of symbolic understanding (which of 2 numerals is bigger), but only for small sets (i.e., 4 or less).[33] The results suggest that children may be able to engage in both symbolic and non-symbolic number sense activities across strands (number, number relations, and number operations) with small set sizes. An intervention in which children engaged in a variety of number sense skills with small set sizes before cycling to a similar sequence with larger set sizes was successful in kindergartners at-risk for later mathematics learning difficulties.[34]

Developmental pathways

Research has revealed patterns of individual differences in early number sense development. There are empirically distinct developmental pathways in number, number relations, and number operations for preschoolers across the school year,[35,36] which predict mathematics achievement in Grades 1 and 3.[35] Low receptive vocabulary[35,36] and visual-spatial working memory skills[34] predict membership in a consistently low developmental pathway, emphasizing the importance of domain-general learning skills for early numeracy development.[37] Contextual factors such as children's home learning environment also relate to individual differences in early numerical development.[38]

Research Gaps

Additional work is needed to consider how the number sense strands of number, number relations, and number operations work together during the early childhood period. Researchers must also consider how set size and level of representation constrain the development of number sense, including for children at-risk for mathematics learning difficulties. Interventions that target and weave together the strands of number sense for children with or at-risk for mathematics learning difficulties should be developed and evaluated through randomized-controlled studies.

Conclusions

Difficulties with mathematics are pervasive and can have lifelong consequences. Foundational number sense skills develop in early childhood and are highly predictive of mathematics achievement and difficulties. The development of number sense depends on level of representation and set size. Research suggests that number sense should be prioritized in preschool and kindergarten to provide a foundation for learning formal arithmetic and developing fluency. Overall, early number sense is critical for setting mathematics trajectories in mathematics throughout elementary school.

Implications for Parents, Service, and Policy

In contemporary educational settings, challenges in learning mathematics may go unnoticed until Grade 4. Early interventions in mathematics are less common than are those for reading, although early screening and multi-tiered intervention programs are growing as we expand our knowledge. Preschools and kindergartens should incorporate mathematics experiences that emphasize instruction in number, number relations and number operations. The curriculum should gradually increase set size and vary type of representation.4,34 It is crucial for curriculum developers in early childhood education to concentrate on fundamental aspects of number sense. By doing so, early interventions can equip all children with the necessary foundations for success in formal mathematics.

References

1. Barbaresi MJ, Katusic SK, Colligan RC, Weaver AL, Jacobsen SJ. Math learning disorder: Incidence in a population-based birth cohort, 1976-1982, Rochester, Minn. *Ambulatory Pediatrics* 2005;5(5):281-289.

2. Reigosa-Crespo V, Valdés-Sosa M, Butterworth B, Estévez N, Rodríguez M, Santos E, Torres P, Suárez R, & Lage A. Basic numerical capacities and prevalence of developmental dyscalculia: the Havana Survey. *Developmental Psychology* 2012;48(1):123-135.

3. Clements DH, Sarama J. Early childhood mathematics learning. In: Lester JFK, ed. *Second handbook of research on mathematics teaching and learning*. New York, NY: Information Age Publishing; 2007:461-555.

4. Cross CT, Woods TA, Schweingruber H, National Research Council, Committee on Early Childhood Mathematics, eds. *Mathematics learning in early childhood: Paths toward excellence and equity*. Washington, DC: National Academies Press; 2009.

5. Sadler PM, Tai RH. The two high-school pillars supporting college science. *Science* 2007;317(5837):457-458.

6. National Mathematics Advisory Panel (NMAP). Foundations for success: The final report of the National Mathematics Advisory Panel. Washington, DC: U.S. Department of Education; 2008.

7. Broer M, Bai Y, Fonseca F. A review of the literature on socioeconomic status and educational achievement. In: *Socioeconomic inequality and educational outcomes: Evidence*

from twenty years of TIMSS. IEA Research for Education, Vol. 5. Cham: Springer; 2019:7-17.

8. Geary DC, Hoard MK, Byrd-Craven J, Nugent L, Numtee C. Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development* 2007;78(4):1343-1359.

9. Berch DB. Making sense of number sense: Implications for children with mathematical disabilities. *Journal of Learning Disabilities* 2005;38(4):333-339.

10. Dehaene S. *The number sense: How the mind creates mathematics.* New York, NY: Oxford University Press; 1997.

11. Feigenson L, Dehaene S, Spelke E. Core systems of number. *TRENDS in Cognitive Sciences* 2004;8(7):307-314.

12. Buijsman S. The representations of the approximate number system. *Philosophical Psychology* 2021;34(2):300-317.

13. Jordan NC, Levine SC. Socioeconomic variation, number competence, and mathematics learning difficulties in young children. *Developmental Disabilities Research Reviews* 2009;15:60-68.

14. Case R, Griffin S. Child cognitive development: The role of central conceptual structures in the development of scientific and social thought. In: Hauert EA, ed. *Developmental psychology: Cognitive, perceptuo-motor, and neurological perspectives*. North-Holland: Elsevier; 1990:193-230.

15. Jordan NC, Devlin BL, Botello M. Core foundations of early mathematics: refining the number sense framework. *Current Opinion in Behavioral Sciences* 2022;46:101181.

16. Jordan NC, Glutting J, Ramineni C. The importance of number sense to mathematics achievement in first and third grades. *Learning and Individual Differences* 2010;20(2):82-88.

17. Duncan GJ, Dowsett CJ, Classens A, Magnuson K, Huston AC, Klebanov P, Pagani LS, Feinstein L, Engel M, Brooks-Gunn J, Sexton H, Duckworth K, Japel C. School readiness and later achievement. *Developmental Psychology* 2007;43(6):1428-1446.

18. Watts TW, Duncan GJ, Siegler RS, Davis-Kean PE. What's past is prologue: Relations between early mathematics knowledge and high school achievement. *Educational Researcher* 2014; 43(7):352-360.

19. Rittle-Johnson B, Fyfe ER, Hofer KG & Farran DC. Early math trajectories: Low-income children's mathematics knowledge from ages 4 to 11. *Child Development* 2017; 88(5):1727-1742.

20. Jordan NC, Kaplan D, Ramineni C, Locuniak MN. Early Math Matters: Kindergarten Number Competence and Later Mathematics Outcomes. *Developmental Psychology* 2009;3(45):850-867.

21. Landerl K, Bevan A, Butterworth B. Developmental dyscalculia and basic numerical capacities: A study of 8– 9-year-old students. *Cognition* 2004;93(2):99-125.

22. Peters, L, de Beeck HO, De Smedt B. Cognitive correlates of dyslexia, dyscalculia and comorbid dyslexia/dyscalculia: Effects of numerical magnitude processing and phonological processing. *Research in Developmental Disabilities* 2020;107:103806.

23. Mazzocco MM, Thompson RE. Kindergarten predictors of math learning disability. *Learning Disabilities Research and Practice* 2005;20(3):142-155.

24. Purpura DJ, Lonigan CJ. Informal numeracy skills: The structure and relations among numbering, relations, and arithmetic operations in preschool. *American Educational Research Journal* 2013; 50(1):178-209.

25. Milburn TF, Lonigan CJ, DeFlorio L, Klein A. Dimensionality of preschoolers' informal mathematical abilities. *Early Childhood Research Quarterly* 2019;47:487-495.

26. Clarke B, Shinn MR. A preliminary investigation into the identification and development of early mathematics curriculum-based measurement. *School Psychology Review* 2004;33(2):234-248.

27. Purpura DJ, Lonigan CJ. Early numeracy assessment: The development of the preschool early numeracy scales. *Early Education and Development* 2015;26(2):286-313.

28. Jordan NC, Klein A, Huang CH. Screener for Early Number Sense. Hammill Institute on Disabilities. (forthcoming).

29. Devlin BL, Jordan NC, Klein A. Predicting mathematics achievement from subdomains of early number competence: Differences by grade and achievement level. *Journal of Experimental Child Psychology* 2022;217:105354.

30. Jordan NC, Kaplan D, Locuniak MN, Ramineni C. Predicting first-grade math achievement from developmental number sense trajectories. *Learning Disabilities Research & Practice* 2007;22(1):36-46.

31. Locuniak MN, Jordan NC. Using kindergarten number sense to predict calculation fluency in second grade. *Journal of Learning Disabilities* 2008;41(5):451-459.

32. Jiminez-Lira CJ, Carver M, Douglas H, LeFevre JA. The integration of symbolic and non-symbolic representations of exact quantity in preschool children. *Cognition* 2017;166:382-397.

33. Hutchison JE, Ansari D, Zheng S, De Jesus S, Lyons IM. The relation between subitizable symbolic and non-symbolic number processing over the kindergarten school year. *Developmental Science* 2020; 23(2):e12884.

34. Dyson N, Jordan NC, Beliakoff A, Hassinger-Das B. A kindergarten number-sense intervention with contrasting practice conditions for low-achieving children. *Journal for Research in Mathematics Education* 2015;46(3):331-370.

35. Bakker M, Torbeyns J, Verschaffel L, De Smedt B. Longitudinal pathways of numerical abilities in preschool: Cognitive and environmental correlates and relation to primary school mathematics achievement. *Developmental Psychology* 2023;59(3):442.

36. Cahoon A, Gilmore C, Simms V. Developmental pathways of early numerical skills during the preschool to school transition. *Learning and Instruction* 2021;75:101484.

37. LeFevre J, Fast L, Skwarchuk SL, Smith-Chant BL, Bisanz J, Kamawar D, Penner-Wilger M. Pathways to mathematics: Longitudinal predictors of performance. *Child Development* 2010 Nov-Dec;81(6):1753-1767.

38. Silver AM, Libertus ME. Environmental influences on mathematics performance in early childhood. *Nature Review Psychology* 2022;1(7):407-418.