



Numeracy

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Table of content

Synthesis	4
<hr/>	
Numerical Knowledge in Early Childhood	7
CATHERINE SOPHIAN, PHD, JUNE 2009	
<hr/>	
Early Predictors of Mathematics Achievement and Mathematics Learning Difficulties	13
¹ NANCY C. JORDAN, PHD, ² BRIANNA L. DEVLIN, PHD, DECEMBER 2023	
<hr/>	
Early Numeracy: The Transition from Infancy to Early Childhood	21
KELLY S. MIX, PHD, JUNE 2010	
<hr/>	
Learning Trajectories in Early Mathematics – Sequences of Acquisition and Teaching	27
DOUGLAS H. CLEMENTS, PHD, JULIE SARAMA, PHD, AUGUST 2023	
<hr/>	
Fostering Early Numeracy in Preschool and Kindergarten	35
ARTHUR J. BAROODY, PHD, JULY 2010	
<hr/>	
Mathematics Instruction for Preschoolers	45
JODY L. SHERMAN-LEVOS, PHD, JULY 2010	
<hr/>	

Synthesis

How important is it?

Numeracy is sometimes defined as understanding how numbers represent specific magnitudes. This understanding is reflected in a variety of skills and knowledge (ex. counting, distinguishing between sets of unequal quantities, operations such as addition and subtraction), and so numeracy often is used to refer to a wide range of number-related concepts and skills. These abilities often emerge in some form well before school entry. The idea of exposing young children to Early Childhood Mathematical Education (ECME) has been around for more than a century, but current discussions revolve around the goals of early training in numeracy and the methods by which these goals should be achieved. Early mathematical learning can and should be integrated in children's everyday activities through encounters with patterns, quantity, and space. Giving children ample and developmentally appropriate opportunities to practice their skills in mathematics, can strengthen the link between children's early abilities in mathematics and the acquisition of mathematical knowledge in school. Unfortunately, children do not all have an equal chance to exercise these skills, hence the importance of ECME. Research on numeracy and early mathematical skills is important to formulate the program and objectives of ECME.

Difficulties in mathematics are relatively common among school-age children. Approximately 1 in 10 children will be diagnosed with a learning disorder related to mathematics during their education. One of the most severe forms is developmental dyscalculia, which refers to an inability to count and tally collections of items and to distinguish numbers from one another.

What do we know?

Basic mathematical knowledge emerges in infancy. At 6 months of age, infants are able to perceive the difference between small sets of elements varying in quantity (2 vs. 3-object sets), and can even distinguish between larger quantities, provided that the ratio between two sets is large enough (ex. 16 vs. 32, but not 8 vs. 12). These preverbal representations become more refined over time, and they form the early, though not sufficient, building blocks of future mathematical learning.

One achievement in numeracy is the acquisition of fact fluency. Fact fluency refers to the knowledge necessary to produce sums and differences in a flexible, timely and accurate manner. In the toddler years, children progressively acquire the requirements for fact fluency, often beginning with intuitive numbers (ex. know the meaning of one, two, three), leading to the ability to recognize that, for example, any set of three elements has a larger count than a set of two elements.

As they get older, children develop more advanced number skills. By age 3, they begin to be proficient in some nonverbal, object-based tasks, such as understanding the process of adding and subtracting, and judging one set as having a larger quantity than a second one. Although preschoolers can match collections of 2, 3, and 4 elements if the objects are of similar size or shape, they still struggle when the objects are highly dissimilar (ex. matching two animal figurines with two black dots). Preschool children are also likely to get easily distracted by superficial features of a set (ex. judging a set of items as having a larger quantity than another equal set because the items are disposed in a longer row). Research is currently under way to determine how knowledge about quantities in infancy is related to preschool numerical competencies and later school achievement.

Although most children can naturally discover mathematical concepts, environment and cultural experiences play a role in advancing children's knowledge about numbers. For instance, language acquisition allows children to solve verbal problems and develop a number sense (ex., understanding cardinality, the total number of elements in a set). Children who lack early experiences with numbers tend to lag behind their peers. For instance, children from economically disadvantaged families tend to display poor numeracy skills early on, and these deficiencies later translate to mathematical difficulties in school. Performance on numerical problems and the kinds of cognitive strategies children use are likely to vary considerably across children. Even the range of one child's responses from one trial to the next can be substantial.

Promoting early competencies in numeracy is important because of its relation to children's mathematical readiness at school entry and beyond. Preschool children who have acquired the ability to count, name numbers, and make distinctions between different quantities tend to perform well on numerical tasks in kindergarten. In addition, children's good numerical abilities predict later school achievement more strongly than their reading, concentration, and socioemotional skills.

What can be done?

Given children's natural dispositions to learn about numbers, they should be encouraged to freely explore and practice their abilities in a range of unstructured activities. These learning experiences should be enjoyable and developmentally appropriate so that children stay engaged in the activity and do not get discouraged. Playing board games and other activities involving experiments with numbers can help children develop their numeracy skills. Materials such as blocks, puzzles, and shapes can also encourage the development of numeracy.

Parents can foster their child's numerical knowledge by creating meaningful experiences with numbers paired with appropriate feedback (ex. asking the child how many feet she has, and using her response to explain why she needs two, and not one shoe). Parents and teachers should also create spontaneous educational moments that encourage the child to think and talk about numbers. Numbers can be introduced in several domains, including play (dice-throwing games), art (drawing a number of stars), and music (keeping a tempo of 2 or 3 beats).

Taking on children's perspective and understanding that their interpretations of mathematical problems are different than adults' are important components of effective education. Teachers need to know that numeracy follows a developmental process, and numerical activities must therefore be designed accordingly. To optimize interventions aimed at numeracy, kindergarten screening should ensure that children can recognize the quantity of small sets of objects (2 and 3) and make the distinction between these and larger sets (4 or 5 objects).

Early interventions in mathematics have important implications for school readiness. A successful ECME program includes a stimulating environment containing objects and toys that encourage mathematical reasoning (ex., table blocks and puzzles), play opportunities where children can develop and expand their natural mathematical abilities on their own, and teachable moments where preschool teachers ask questions about children's mathematical discoveries.

Numerical Knowledge in Early Childhood

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Introduction

Research on the numerical knowledge of young children has grown rapidly in recent years. This research encompasses a wide range of abilities and concepts, from infants' ability to discriminate between collections containing different numbers of elements^{1,2} to preschoolers' understanding of number words^{3,4} and counting,^{5,6,7} and their grasp of the inverse relation between addition and subtraction.^{8,9}

Subject

Research on young children's numerical knowledge provides an important foundation for the formulation of standards for early childhood education¹⁰ and for the design of early childhood mathematics curricula.^{11,12,13} Further, the mathematics knowledge that children acquire before they begin formal schooling has important ramifications for school performance and future career options.¹⁴ An analysis of predictors of academic achievement, based on six longitudinal data sets, found that children's math skills at school entry predicted subsequent school performance more strongly than did early reading skills, attentional skills or socioemotional skills.¹⁵

Problems

Fundamentally, numeracy entails understanding numbers as representations of a particular kind of magnitude. Correspondingly, understanding the development of numeracy in early childhood entails understanding both how children come to understand the basic quantitative relations that numbers share with other kinds of quantities and how they come to understand the aspects of number that distinguish it from other kinds of quantities.

Research Context

Piaget's classic research on logico-mathematical development investigated children's understanding of general properties of quantity such as seriation and the conservation of

equivalence relations under certain kinds of transformations.¹⁶ His view, however, was that this kind of knowledge emerges only with the acquisition of concrete-operational thinking, around 5-7 years of age. Subsequent researchers¹⁷ undertook to demonstrate that younger children have considerably more numerical knowledge than Piaget recognized; and contemporary research provides evidence of a wide range of early numerical abilities.¹⁸

Key Research Questions

An influential but controversial claim in current research literature on early numerical abilities holds that the brain is “hard wired” for number.^{19,20} This idea is often supported by evidence of numerical discrimination by human infants and by animals.²¹ Critics of innatist (philosophical doctrine that holds that the mind is born with ideas/knowledge) accounts of numerical knowledge, however, note the pervasiveness of developmental change in numerical reasoning,²² the slow differentiation of number from other quantitative dimensions,²³ and the contextualized nature of early numerical knowledge.²⁴ Further, accumulating evidence indicates that language²⁴ and other cultural products and practices^{25,26} make enormous contributions to young children’s acquisition of numerical knowledge.

Recent Research Results

Numerical knowledge in infancy

One of the most active areas of current research concerns the numerical abilities of infants. Kobayashi, Hiraki and Hasegawa¹ used discrepancies between visual and auditory information about the number of items in a collection to test for numerical discrimination in six-month-olds. They showed infants objects that made a sound when dropped onto a surface, and then dropped two or three of the objects behind a screen so that the infants heard the tone each item made but could not see the items. They then removed the screen to reveal either the correct number of objects or a different number (3 if there had been 2 tones, and vice versa). Infants looked longer when the number of items revealed did not match the number of tones, indicating that they were able to distinguish between two and three items. Other research indicates that six-month-old infants can also discriminate between larger numerical quantities, provided the numerical ratio between them is large. Six-month-old infants discriminate between 4 vs. 8²⁷ and even 16 vs. 32.²⁸ When the contrast is reduced (for example, 8 vs. 12), however, six-month-old infants fail²⁹ but older ones succeed.² Thus, infants become able to make finer numerical discriminations as they

get older.

Young children's knowledge about numerical relations

Because numbers represent a kind of magnitude, a fundamental aspect of numerical knowledge pertains to equal, less-than and greater-than relations between numerical quantities.³⁰ Somewhat surprisingly, in light of the infancy findings, it is a significant developmental achievement for preschool children to compare sets numerically, particularly when that entails disregarding other differences between the sets.

For example, Mix³¹ studied the ability of three-year-olds to numerically match a set of 2, 3 or 4 black dots. This task was easy when the manipulatives children were given were perceptually similar to the dots they were to match (e.g., black disks, or red shells about the same size as the dots). However, children's performance dropped when the manipulatives contrasted perceptually with the dots (e.g., lion figurines or heterogeneous objects).

Muldoon, Lewis, and Francis⁷ assessed four-year-olds' ability to evaluate the numerical relation between two rows of blocks (with 6-9 items per row) in the face of misleading length cues, that is, when two unequal-length rows contained the same number of items, or two equal-length rows contained different numbers of items. Most children relied on length comparisons rather than on counting the items to compare the rows. However, a three-session training procedure led to better performance, particularly among children who, as part of the training, were asked to explain why the rows were in fact numerically equal or unequal (as indicated by the experimenter).

Research Gaps

While experimental data concerning early numeracy is accumulating rapidly, the absence of theoretical accounts that incorporate the full range of empirical results limits our understanding of how the diverse findings already obtained fit together and what issues remain to be resolved. In the infancy literature, for example, competing accounts of early numerical abilities have generated much research in the past few years, yet the findings have not lessened the theoretical controversy. In advancing theoretical conclusions, researchers need to be cognizant of the entire corpus of findings, and their theories need to be formulated precisely enough that they can be differentiated empirically.

In addition, researchers need to gather better information about the processes that lead to advances in early numeracy knowledge. We know that young children’s performance is affected by contextual variables ranging from culture and social class³² to patterns of parent-child^{33,34} and teacher-child³⁵ interaction. As yet, however, we have only small pieces of information, mostly from experimental training studies^{7,25,36} about how particular experiences alter children’s numerical thinking. Research that provides converging data about (a) young children’s everyday numerical experiences, and how they vary with the age of the child, and (b) the experimental effects of those kinds of experiences on children’s thinking, would be especially helpful.

Conclusions

The available research on young children’s developing knowledge about number supports four generalizations that have important implications for policy and practice. First, numerical development is multifaceted. Early childhood numeracy encompasses much more than counting and knowing some elementary arithmetic facts. Second, notwithstanding the number-related abilities evidenced even by infants, age-related change is pervasive. In age group comparisons, the older children nearly always perform better. Third, variability is pervasive. Individual children vary in their performance across different numerical tasks,³⁷ in their engagement in particular sorts of numerical reasoning across different contexts,³ and even in their trial-to-trial responses within a single task.^{5,38} Finally, children’s progress in acquiring numerical knowledge is highly malleable. It is influenced by informal activities such as playing board games,²⁵ by experimental activities designed to illuminate numerical relationships,^{7,36} and by variations in the ways in which parents^{33,34} and teachers³⁵ talk to children about numbers.

Implications

An important contribution that research on early childhood numeracy can make to policy and practice is to inform the goals we set for early mathematics instruction. Just as numerical development in early childhood is multi-faceted, the goals of early childhood instructional programs should be much broader than enhancing children’s counting skills or teaching them some basic arithmetic facts. Numbers, like other kinds of magnitudes, are characterized by relations of equality and inequality. At the same time, they differ from other kinds of magnitudes in that they are based on the partitioning of an overall quantity into units. Instructional activities that encourage children to think about relationships between quantities and effects of transformations such as partitioning, grouping, or rearranging those relationships may be helpful

in advancing children’s understanding of these ideas. The variability and malleability of young children’s numerical thinking indicate the potential for early childhood instructional programs to contribute substantially to children’s growing knowledge about numbers.

References

1. Kobayashi T, Hiraki K, Hasegawa T. Auditory-visual intermodal matching of small numerosities in 6-month-old infants. *Developmental Science* 2005;8(5):409-419.
2. Xu F, Arriaga RI. Number discrimination in 10-month-olds. *British Journal of Developmental Psychology* 2007;25(1):103-108.
3. Mix KS. How Spencer made number: First uses of the number words. *Journal of Experimental Child Psychology* 2009;102(4):427-444.
4. Sarnecka BW, Lee MD. Levels of number knowledge in early childhood. *Journal of Experimental Child Psychology* 2009;103(3):325-337.
5. Chetland E, Fluck M. Children’s performance on the ‘give-x’ task: A microgenetic analysis of ‘counting’ and ‘grabbing’ behaviour. *Infant and Child Development* 2007;16(1):35-51.
6. Le Corre M, Carey S. One, two, three, four, nothing more: an investigation of the conceptual sources of the verbal counting principles. *Cognition* 2007;105(2):395-438.
7. Muldoon K, Lewis C, Francis B. Using cardinality to compare quantities: The role of social-cognitive conflict in the development of basic arithmetical skills. *Developmental Science* 2007;10(5):694-711.
8. Canobi KH, Bethune NE. Number words in young children’s conceptual and procedural knowledge of addition, subtraction and inversion. *Cognition* 2008;108(3):675-686.
9. Sherman J, Bisanz J. Evidence for use of mathematical inversion by three-year-old children. *Journal of Cognition and Development* 2007;8(3):333-344.
10. Clements DH, Sarama J, DiBiase AM, eds. *Engaging young children in mathematics: Standards for early childhood mathematics education*. Mahwah, N.J.: Lawrence Erlbaum Associates; 2004.
11. Clements DH, Sarama J. Experimental evaluation of the effects of a research-based preschool mathematics curriculum. *American Educational Research Journal* 2008; 45(2):443-494.
12. Griffin S, Case R. Re-thinking the primary school math curriculum: An approach based on cognitive science. *Issues in Education* 1997;3(1):1-49.
13. Starkey P, Klein A, Wakeley A. Enhancing young children’s mathematical knowledge through a pre-kindergarten mathematics intervention. *Early Childhood Research Quarterly* 2004;19(1):99-120.
14. National Mathematics Advisory Panel. *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC.: U. S. Department of Education; 2008.
15. Duncan GJ, Dowsett CJ, Claessens A, Magnuson K, Huston AC, Klebanov P, Pagani LS, Feinstein L, Engel M, Brooks-Gunn J, Sexton H, Duckworth K, Japel C. School readiness and later achievement. *Developmental Psychology*. 2007;43(6):1428 - 46.
16. Piaget J. *The child's conception of number*. Gattegno C, Hodgson FM, trans. New York, NY: Norton; 1952.
17. Gelman R, Gallistel CR. *The child's understanding of number*. Cambridge, MA: Harvard University Press; 1978.
18. Geary DC. Development of mathematical understanding. In: Damon W, ed. *Handbook of child psychology*. 6th ed. New York, NY: John Wiley & Sons; 2006:777-810. Khun D, Siegler RS, eds. *Cognition, perception, and language*. Vol. 2.
19. Butterworth B. *The mathematical brain*. New York, NY: Macmillan; 1999.

20. Dehaene S. *The number sense: How the mind creates mathematics*. Oxford, UK: Oxford University Press; 1997
21. Feigenson L, Dehaene S, Spelke E. Core systems of number. *Trends in Cognitive Sciences* 2004;8(3):307-314.
22. Sophian C. Beyond competence: The significance of performance for conceptual development. *Cognitive Development* 1997;12(3):281-303.
23. Sophian C. *The origins of mathematical knowledge in childhood*. New York, NY: Lawrence Erlbaum Associates; 2007.
24. Mix KS, Sandhofer CM, Baroody AJ. Number words and number concepts: The interplay of verbal and nonverbal quantification in early childhood. In: RV Kail, ed. *Advances in child development and behavior*. vol. 33. New York, NY: Academic Press; 2005:305-346.
25. Ramani GB, Siegler RS. Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development* 2008;79(2):375-394.
26. Schliemann AD, Carraher DW. The evolution of mathematical reasoning: Everyday versus idealized understandings. *Developmental Review* 2002;22(2):242-266.
27. Xu F. Numerosity discrimination in infants: Evidence for two systems of representation. *Cognition* 2003;89(1):B15-B25
28. Xu F, Spelke ES, Goddard S. Number sense in human infants. *Developmental Science* 2005;8(1):88-101.
29. Xu F, Spelke ES. Large-number discrimination in 6-month-old infants. *Cognition* 2000;74(1):B1-B11.
30. Davydov VV. Logical and psychological problems of elementary mathematics as an academic subject. In: Kilpatrick J, Wirsup I, Begle EG, Wilson JW, eds. *Soviet studies in the psychology of learning and teaching mathematics*. Chicago, Ill: University of Chicago Press; 1975: 55-107. Steffe LP, ed. *Children's capacity for learning mathematics*. Vol. 7.
31. Mix KS. Surface similarity and label knowledge impact early numerical comparisons. *British Journal of Developmental Psychology* 2008;26(1):1-11.
32. Starkey P, Klein A. Sociocultural influences on young children's mathematical knowledge. In: Saracho ON, Spodek B, eds. *Contemporary perspectives on mathematics in early childhood education*. Charlotte, NC: IAP/Information Age Pub.; 2008:253-276.
33. Blevins-Knabe B, Musun-Miller L. Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development and Parenting* 1996;5(1):35-45.
34. Lefevre J, Clarke T, Stringer AP. Influences of language and parental involvement on the development of counting skills: Comparisons of French- and English-speaking Canadian children. *Early Child Development and Care* 2002;172(3):283-300.
35. Klibanoff RS, Levine SC, Huttenlocher J, Vasilyeva M, Hedges LV. Preschool children's mathematical knowledge: The effect of teacher "math talk." *Developmental Psychology* 2006;42(1):59-69.
36. Sophian C, Garyantes D, Chang C. When three is less than two: Early developments in children's understanding of fractional quantities. *Developmental Psychology* 1997;33(5):731-744.
37. Dowker A. Individual differences in numerical abilities in preschoolers. *Developmental Science* 2008;11(5):650-654.
38. Siegler RS. How does change occur: A microgenetic study of number conservation. *Cognitive Psychology* 1995;28(3):225-273.

Early Predictors of Mathematics Achievement and Mathematics Learning Difficulties

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Introduction

Mathematics difficulties are widespread. Up to 10% of students are diagnosed with a learning disability in mathematics at some point in their school careers.^{1,2} Many more learners struggle in mathematics without a formal diagnosis. Mathematics difficulties are persistent, and students who have difficulties may never catch up to their normally achieving peers without intervention.

Subject

Foundations for mathematics achievement are established before children enter primary school.^{3,4} Identification of key predictors of mathematics outcomes provides support for screening, intervention and progress monitoring before children fall seriously behind in school.

Problem

The consequences of poor mathematics achievement are serious for everyday functioning, educational attainment, and career advancement.⁵ Mathematics competence is necessary for entry into STEM (science, technology, engineering and mathematics) disciplines in college and for STEM-related occupations.⁶ There are large group differences in mathematics achievement related to socioeconomic status⁷ as well as individual differences in general learning abilities.⁸ These disparities are already present in early childhood and increase over the course of schooling.

Research Context

Longitudinal studies of characteristics of children with mathematics difficulties have identified important targets for intervention. Most children enter school with *number sense* that is relevant to learning school mathematics. Preverbal components of number (e.g., perceiving exact representations of small sets of objects and approximate representations of larger sets) develop in infancy.^{9,10,11,12} Although these primary foundations are thought to underlie learning of conventional

mathematics skills, they are not sufficient. Most children with difficulties in mathematics show weaknesses in number sense related to knowledge of number, number relations, and number operations^{4,13} – strands of number sense are malleable and influenced by experience.¹⁴ Early number relates to knowledge of oral and written number and counting concepts, such as one-to-one correspondence and cardinality. Number relations involve understanding of numerical magnitudes on the number line. Number operations relates to transforming quantities through addition and subtraction.^{4,15}

Key Research Questions

Early competencies that are aligned with the mathematics children are required to do in school are most predictive of mathematics achievement and difficulties.¹⁶ Assessment tools and interventions need to be refined to help children develop key number sense concepts. Identifying pathways and factors related to number sense development is needed to guide the creation of early childhood interventions for those at-risk for developing mathematics learning difficulties in school.

Recent Research Results

Early number sense sets children’s achievement trajectories in mathematics.^{16,17,18,19,20} Mathematics difficulties and disabilities have their roots in poorly developed number sense.^{21,22,23} Children with developmental dyscalculia, a severe form of mathematics disability, are characterized by deficits in counting and enumerating sets of objects and in recognizing and comparing numbers.²¹ Such deficits lead to poor arithmetic fluency, a hallmark skill in the primary grades.

Number sense as a predictor of later mathematics achievement and difficulty

Studies provide empirical evidence of a multifactor early number sense model consisting of specific strands of number, number relations, and number operations understanding.^{24,25} Early screening tools developed using this model accurately identify children at risk for mathematics learning difficulty and disability.^{26,27,28} Predictive relations may differ by level of number sense in preschool. Number strand skills predict later mathematics achievement for children with low and intermediate achievement, but not high achievement; number relations strand skills predict later achievement for children of all math achievement levels; number operations strand skills predict later math achievement for children with intermediate and high achievement, but not low achievement.²⁹

Low-income children enter kindergarten well behind their middle-income peers on most symbolic numeracy indicators, and this gap does not narrow during the school year.¹³ Longitudinal studies over multiple time points, from the beginning of kindergarten through the end of Grade 3, suggest that foundational number sense supports the learning of complex mathematics associated with computation as well as applied problem solving and fluency.^{16,20,30,31} The low mathematics achievement of high-risk, low-income students is mediated by kindergarten number sense. Because early number competencies are achievable in most children⁴ their intermediate effects provide clear directions for early intervention

Type of quantity representation and set size

The size of a quantity or set along with the way it is presented to a child (e.g., non-symbolic or symbolic) affects children's reasoning about numbers. Children's ability to map written numerals to the quantities they represent is critical for learning more complex number sense skills.³² Non-symbolic quantity representations (e.g., which of 2 sets of dots has more, without counting) scaffold the development of symbolic understanding (which of 2 numerals is bigger), but only for small sets (i.e., 4 or less).³³ The results suggest that children may be able to engage in both symbolic and non-symbolic number sense activities across strands (number, number relations, and number operations) with small set sizes. An intervention in which children engaged in a variety of number sense skills with small set sizes before cycling to a similar sequence with larger set sizes was successful in kindergartners at-risk for later mathematics learning difficulties.³⁴

Developmental pathways

Research has revealed patterns of individual differences in early number sense development. There are empirically distinct developmental pathways in number, number relations, and number operations for preschoolers across the school year,^{35,36} which predict mathematics achievement in Grades 1 and 3.³⁵ Low receptive vocabulary^{35,36} and visual-spatial working memory skills³⁴ predict membership in a consistently low developmental pathway, emphasizing the importance of domain-general learning skills for early numeracy development.³⁷ Contextual factors such as children's home learning environment also relate to individual differences in early numerical development.³⁸

Research Gaps

Additional work is needed to consider how the number sense strands of number, number relations, and number operations work together during the early childhood period. Researchers must also consider how set size and level of representation constrain the development of number sense, including for children at-risk for mathematics learning difficulties. Interventions that target and weave together the strands of number sense for children with or at-risk for mathematics learning difficulties should be developed and evaluated through randomized-controlled studies.

Conclusions

Difficulties with mathematics are pervasive and can have lifelong consequences. Foundational number sense skills develop in early childhood and are highly predictive of mathematics achievement and difficulties. The development of number sense depends on level of representation and set size. Research suggests that number sense should be prioritized in preschool and kindergarten to provide a foundation for learning formal arithmetic and developing fluency. Overall, early number sense is critical for setting mathematics trajectories in mathematics throughout elementary school.

Implications for Parents, Service, and Policy

In contemporary educational settings, challenges in learning mathematics may go unnoticed until Grade 4. Early interventions in mathematics are less common than are those for reading, although early screening and multi-tiered intervention programs are growing as we expand our knowledge. Preschools and kindergartens should incorporate mathematics experiences that emphasize instruction in number, number relations and number operations. The curriculum should gradually increase set size and vary type of representation.^{4,34} It is crucial for curriculum developers in early childhood education to concentrate on fundamental aspects of number sense. By doing so, early interventions can equip all children with the necessary foundations for success in formal mathematics.

References

1. Barbaresi MJ, Katusic SK, Colligan RC, Weaver AL, Jacobsen SJ. Math learning disorder: Incidence in a population-based birth cohort, 1976-1982, Rochester, Minn. *Ambulatory Pediatrics* 2005;5(5):281-289.

2. Reigosa-Crespo V, Valdés-Sosa M, Butterworth B, Estévez N, Rodríguez M, Santos E, Torres P, Suárez R, & Lage A. Basic numerical capacities and prevalence of developmental dyscalculia: the Havana Survey. *Developmental Psychology* 2012;48(1):123-135.
3. Clements DH, Sarama J. Early childhood mathematics learning. In: Lester JFK, ed. *Second handbook of research on mathematics teaching and learning*. New York, NY: Information Age Publishing; 2007:461-555.
4. Cross CT, Woods TA, Schweingruber H, National Research Council, Committee on Early Childhood Mathematics, eds. *Mathematics learning in early childhood: Paths toward excellence and equity*. Washington, DC: National Academies Press; 2009.
5. Sadler PM, Tai RH. The two high-school pillars supporting college science. *Science* 2007;317(5837):457-458.
6. National Mathematics Advisory Panel (NMAP). Foundations for success: The final report of the National Mathematics Advisory Panel. Washington, DC: U.S. Department of Education; 2008.
7. Broer M, Bai Y, Fonseca F. A review of the literature on socioeconomic status and educational achievement. In: *Socioeconomic inequality and educational outcomes: Evidence from twenty years of TIMSS. IEA Research for Education, Vol. 5*. Cham: Springer; 2019:7-17.
8. Geary DC, Hoard MK, Byrd-Craven J, Nugent L, Numtee C. Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development* 2007;78(4):1343-1359.
9. Berch DB. Making sense of number sense: Implications for children with mathematical disabilities. *Journal of Learning Disabilities* 2005;38(4):333-339.
10. Dehaene S. *The number sense: How the mind creates mathematics*. New York, NY: Oxford University Press; 1997.

11. Feigenson L, Dehaene S, Spelke E. Core systems of number. *TRENDS in Cognitive Sciences* 2004;8(7):307-314.
12. Buijsman S. The representations of the approximate number system. *Philosophical Psychology* 2021;34(2):300-317.
13. Jordan NC, Levine SC. Socioeconomic variation, number competence, and mathematics learning difficulties in young children. *Developmental Disabilities Research Reviews* 2009;15:60-68.
14. Case R, Griffin S. Child cognitive development: The role of central conceptual structures in the development of scientific and social thought. In: Hauert EA, ed. *Developmental psychology: Cognitive, perceptuo-motor, and neurological perspectives*. North-Holland: Elsevier; 1990:193-230.
15. Jordan NC, Devlin BL, Botello M. Core foundations of early mathematics: refining the number sense framework. *Current Opinion in Behavioral Sciences* 2022;46:101181.
16. Jordan NC, Glutting J, Ramineni C. The importance of number sense to mathematics achievement in first and third grades. *Learning and Individual Differences* 2010;20(2):82-88.
17. Duncan GJ, Dowsett CJ, Classens A, Magnuson K, Huston AC, Klebanov P, Pagani LS, Feinstein L, Engel M, Brooks-Gunn J, Sexton H, Duckworth K, Japel C. School readiness and later achievement. *Developmental Psychology* 2007;43(6):1428-1446.
18. Watts TW, Duncan GJ, Siegler RS, Davis-Kean PE. What's past is prologue: Relations between early mathematics knowledge and high school achievement. *Educational Researcher* 2014; 43(7):352-360.
19. Rittle-Johnson B, Fyfe ER, Hofer KG & Farran DC. Early math trajectories: Low-income children's mathematics knowledge from ages 4 to 11. *Child Development* 2017; 88(5):1727-1742.

20. Jordan NC, Kaplan D, Ramineni C, Locuniak MN. Early Math Matters: Kindergarten Number Competence and Later Mathematics Outcomes. *Developmental Psychology* 2009;3(45):850-867.
21. Landerl K, Bevan A, Butterworth B. Developmental dyscalculia and basic numerical capacities: A study of 8- 9-year-old students. *Cognition* 2004;93(2):99-125.
22. Peters, L, de Beeck HO, De Smedt B. Cognitive correlates of dyslexia, dyscalculia and comorbid dyslexia/dyscalculia: Effects of numerical magnitude processing and phonological processing. *Research in Developmental Disabilities* 2020;107:103806.
23. Mazzocco MM, Thompson RE. Kindergarten predictors of math learning disability. *Learning Disabilities Research and Practice* 2005;20(3):142-155.
24. Purpura DJ, Lonigan CJ. Informal numeracy skills: The structure and relations among numbering, relations, and arithmetic operations in preschool. *American Educational Research Journal* 2013; 50(1):178-209.
25. Milburn TF, Lonigan CJ, DeFlorio L, Klein A. Dimensionality of preschoolers' informal mathematical abilities. *Early Childhood Research Quarterly* 2019;47:487-495.
26. Clarke B, Shinn MR. A preliminary investigation into the identification and development of early mathematics curriculum-based measurement. *School Psychology Review* 2004;33(2):234-248.
27. Purpura DJ, Lonigan CJ. Early numeracy assessment: The development of the preschool early numeracy scales. *Early Education and Development* 2015;26(2):286-313.
28. Jordan NC, Klein A, Huang CH. Screener for Early Number Sense. Hammill Institute on Disabilities. (forthcoming).
29. Devlin BL, Jordan NC, Klein A. Predicting mathematics achievement from subdomains of early number competence: Differences by grade and achievement level. *Journal of Experimental Child Psychology* 2022;217:105354.

30. Jordan NC, Kaplan D, Locuniak MN, Ramineni C. Predicting first-grade math achievement from developmental number sense trajectories. *Learning Disabilities Research & Practice* 2007;22(1):36-46.
31. Locuniak MN, Jordan NC. Using kindergarten number sense to predict calculation fluency in second grade. *Journal of Learning Disabilities* 2008;41(5):451-459.
32. Jiminez-Lira CJ, Carver M, Douglas H, LeFevre JA. The integration of symbolic and non-symbolic representations of exact quantity in preschool children. *Cognition* 2017;166:382-397.
33. Hutchison JE, Ansari D, Zheng S, De Jesus S, Lyons IM. The relation between subitizable symbolic and non-symbolic number processing over the kindergarten school year. *Developmental Science* 2020; 23(2):e12884.
34. Dyson N, Jordan NC, Beliakoff A, Hassinger-Das B. A kindergarten number-sense intervention with contrasting practice conditions for low-achieving children. *Journal for Research in Mathematics Education* 2015;46(3):331-370.
35. Bakker M, Torbeyns J, Verschaffel L, De Smedt B. Longitudinal pathways of numerical abilities in preschool: Cognitive and environmental correlates and relation to primary school mathematics achievement. *Developmental Psychology* 2023;59(3):442.
36. Cahoon A, Gilmore C, Simms V. Developmental pathways of early numerical skills during the preschool to school transition. *Learning and Instruction* 2021;75:101484.
37. LeFevre J, Fast L, Skwarchuk SL, Smith-Chant BL, Bisanz J, Kamawar D, Penner-Wilger M. Pathways to mathematics: Longitudinal predictors of performance. *Child Development* 2010 Nov-Dec;81(6):1753-1767.
38. Silver AM, Libertus ME. Environmental influences on mathematics performance in early childhood. *Nature Review Psychology* 2022;1(7):407-418.

Early Numeracy: The Transition from Infancy to Early Childhood

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Introduction

Number concepts emerge before formal schooling. Preschool children exhibit verbal skills, such as counting, and basic concepts of equivalence, ordinality, and quantitative transformation. Although researchers agree that these abilities exist in early childhood, they continue to debate when, and by what mechanisms, these abilities emerge. In other words, what are the developmental origins of early numeracy?

Subject

Research on numeracy has traditionally focused on verbal counting. However, the notion that numeracy might emerge in infancy and toddlerhood shifted the focus toward nonverbal abilities. This shift expanded the range of behaviours included in early numeracy, a change that has direct implications for early childhood education and assessment. This shift also raised questions about the developmental origins of math disabilities and gaps in mathematical achievement, such as those associated with difference socioeconomic groups.

Problems

Current developmental accounts differ in the weight given to nonverbal versus verbal representations.

Some argue the core conceptual structure for number is inborn and takes the form of a nonverbal representation that is similar to verbal counting.^{1,2,3} On this view, a major developmental achievement is mapping verbal number words onto their nonverbal referents.

Others claim innate processes contribute to numerical development but do not constitute a complete conceptual system for number.^{4,5} These accounts incorporate both preverbal counting and a second representational format based on object tracking. They characterize verbal counting

as a conceptual catalyst that permits integration of the two nonverbal representations,⁵ thereby transcending their inherent limitations and achieving a true concept of number.⁴

Yet other accounts incorporate object-based representations but claim that these representations develop during early childhood.⁶ On this view, object representations of number are not precise, even for small sets. Instead, they are thought to approximate number with increasing accuracy due to (1) age-related increases in working memory capacity, and (2) interactions between partial knowledge of the number words and recognition of small numerosities in specific contexts.^{6,7,8}

Some argue that number concepts are extracted from the counting system itself, without support from nonverbal representations. Studies have shown that children do not understand counting principles until they have mastered counting procedures.^{9,10} It has also been argued that children cannot connect labels for small sets to the conventional counting system because they cannot pick out the natural number sequence from other sequences.¹¹

Research Context

Because research has focused on the emergence of verbal numeracy within a nonverbal conceptual base, existing experiments include a mixture of verbal and nonverbal methods. In the verbal realm, investigators measure various subcomponents of counting (e.g., asking children to recite the count list, count a set of objects, or name a set's cardinality). In the nonverbal realm, investigators use object-based tasks that do not require verbal counting. With very young children and infants, looking time procedures (e.g., habituation) and reaching tasks are common.

Key Research Questions

A major aim has been to describe the numerical sensitivity of infants and very young children. Researchers want to know how much children understand about number before acquiring conventional skills. The specific profile of nonverbal strengths and weaknesses is sometimes used to argue for a particular developmental account. Another major research goal is describing the emergence of verbal numeracy in great detail. In this research, the potential interactions between verbal and nonverbal numeracy is carefully considered.

Recent Research Results

Numerical sensitivity in infants

Early habituation research indicated infants could discriminate between small sets of objects. For example, when babies were shown a series of object sets equated for number (e.g., two), but varying in color, shape, and position, their looking time gradually decreased. When a new number of objects was shown (e.g., three), looking times increased, suggesting that infants detected the change in number.^{12,13} Similar experiments have suggested infants can discriminate large sets of items in both visual and auditory displays,^{14,15} perform simple calculations over objects,³ and detect numerical relations across modalities.^{16,17}

Nonverbal measures in early childhood

Children perform object-based number tasks much earlier than they demonstrate similar understandings in verbal tasks. For example, preschoolers solve simple object-based addition and subtraction problems (e.g., $2 + 2$) years before they can solve analogous verbal problems.^{6,8,18} Similarly, children judge ordinality and equivalence in forced choice tasks much earlier than they can compare the same sets verbally, via counting.^{6,19,20,21,22,23,24} Competence on nonverbal measures emerges between 2½ and 3 years of age.

Development of verbal counting

Verbal counting encompasses three major subskills. First, children must learn the sequence of count words. The first 10 count words are usually memorized by age 3 years.^{25,26} Children learn to generate numbers using the decade structure (teens, twenties, etc.) around 6 years of age. Second, young counters must coordinate words and objects, such that each item in a set is tagged once and only once. Children make many errors as they discover and master tagging procedures, peaking in frequency between 36 and 42 months of age.²⁵ Third, children learn that the last word in a count represents its cardinal value (e.g., when you count, “1-2-3,” you have three things). Interestingly, children gain this insight before mastering verbal counting procedures, which suggests they access the cardinal word principle via experiences with small sets.^{4,25,26,27,28,29} In fact, small set sizes (i.e., 1-3) may provide the only context for discovering the cardinal word principle because these can be both counted and labeled without counting.^{4,26,27,28,29,30,31,32,33}

Research Gaps

A persistent problem has been reconciling infants’ apparent precocity for number with the struggles exhibited by preschool children on similar tasks. For example, if infants can represent and compare large object sets as some have claimed,¹⁵ why can’t preschool children match large

sets until after they have learned to count?^{34,35} Such discrepancies have fueled vigorous debate about the meaning of the infant work, and articulating these literatures remains a significant challenge. For example, researchers have only begun to ask whether infant sensitivity to quantity is connected to preschool numeracy and, likewise, whether preschool numeracy is connected to subsequent achievement in school mathematics.³⁶

Another unexplored question is how children coordinate discrete and continuous quantity. Infants' perception of continuous amount is well established. Some have argued use of continuous amount actually explains infants' performance on numerical tasks.^{37,38} Regardless of whether infants process continuous quantity, discrete number or both, research is needed to determine what causes them to shift attention from one type of quantification to the other, as well as the developmental changes that occur as children learn how continuous and discrete quantity are related (e.g., size does not affect counting, unless you are counting measurement units).

Finally, much remains to be learned about the interactions between nonverbal quantification and verbal counting. Some contend that whatever preverbal infants can do or understand is necessarily innate because it emerges without verbal input.⁴ However, others have argued that even infants who do not speak about number themselves have nonetheless been exposed to number language, and thus it is not clear that infant competencies are either nonverbal or innate.³⁹ A related issue is how children acquire the meanings of the number words and the extent to which this relies on a nonverbal foundation. Current research is also exploring whether acquisition of the plural mediates these interactions.⁴⁰

Conclusions

Evidence of numerical competence in infants has raised interesting questions about the origins of numeracy and the conceptual resources young children use to acquire verbal counting. However, further research is needed to reveal what this infant competence entails and precisely how it connects to subsequent nonverbal and verbal development.

References

1. Dehaene S. *The number sense: How the mind creates mathematics*. Oxford, England: Oxford University Press; 1997.
2. Gallistel CR, Gelman R. Preverbal and verbal counting and computation *Cognition* 1992;44: 43-74.
3. Wynn K. Origins of numerical knowledge. *Mathematical cognition* 1995;1:35-60.
4. Carey S. Whorf versus continuity theorists: Bringing data to bear on the debate. In: Bowerman M, Levinson SC, eds. *Language acquisition and conceptual development*. New York, NY: Cambridge University Press: 2001;185-214.

5. Spelke E. What makes us smart? Core knowledge and natural language. In: Gentner D, Goldin-Meadow S, eds. *Language in mind*. Cambridge, MA: MIT Press; 2003.
6. Huttenlocher J, Jordan N, Levine SC. A mental model for early arithmetic. *Journal of Experimental Psychology: General* 1994;123:284-296.
7. Mix KS, Sandhofer CM., Baroody A. Number words and number concepts: The interplay of verbal and nonverbal processes in early quantitative development. In: Kail RV, ed. *Advances in Child Development and Behavior*. New York, NY: Academic Press; 2005: 305-345.
8. Rasmussen C, Bisanz J. Representation and working memory in early arithmetic. *Journal of Experimental Child Psychology* 2005; 91:137-157.
9. Briars DJ, Siegler RS. A featural analysis of preschoolers' counting knowledge. *Developmental Psychology* 1984;20:607-618.
10. Frye D, Braisby N, Lowe J, Maroudas C, Nicholls J. Young children's understanding of counting and cardinality. *Child Development* 1989;60:1158-1171.
11. Rips LJ, Asmuth J, Bloomfield A. Giving the boot to the bootstrap: How not to learn natural numbers. *Cognition* 2006;101:B51-B60.
12. Antell S, Keating DP. Perception of numerical invariance in neonates. *Child Development* 1983;54:695-701.
13. Strauss MS, Curtis LE. Infant perception of numerosity. *Child Development* 1981;52:1146-1152.
14. Lipton JS, Spelke ES. Origins of number sense: Large number discrimination in human infants. *Psychological Science* 2003;14: 396-401.
15. Xu F, Spelke ES. Large number discrimination in 6-month-old infants. *Cognition* 2000;74: B1-B11.
16. Starkey P, Spelke ES, Gelman R. Numerical abstraction by human infants. *Cognition* 1990;36:97-127.
17. Jordan KE, Suanda SH, Brannon EM. Intersensory redundancy accelerates preverbal numerical competence. *Cognition* 2008;108: 210-221.
18. Levine SC, Jordan NC, Huttenlocher J. Development of calculation abilities in young children. *Journal of Experimental Child Psychology* 1992;53:72-103.
19. Cantlon J, Fink R, Safford K, Brannon EM. Heterogeneity impairs numerical matching but not numerical ordering in preschool children. *Developmental Science* 2007;10:431-440.
20. Mix KS. Preschoolers' recognition of numerical equivalence: Sequential sets. *Journal of Experimental Child Psychology* 1999;74:309-322.
21. Mix KS. Similarity and numerical equivalence: Appearances count. *Cognitive Development* 1999;14:269-297.
22. Mix KS. The construction of number concepts. *Cognitive Development* 2002;17:1345-1363.
23. Mix KS. Children's equivalence judgments: Crossmapping effects. *Cognitive Development* 2008;23:191-203.
24. Mix KS, Huttenlocher J, Levine SC. Do preschool children recognize auditory-visual numerical correspondences? *Child Development* 1996; 67:1592-1608.
25. Fuson KC. *Children's counting and conceptions of number*. New York, NY: Springer-Verlag; 1988.
26. Bermejo V. Cardinality development and counting. *Developmental Psychology* 1996;32:263-268.
27. Mix KS. How Spencer made number: First uses of the number words. *Journal of Experimental Child Psychology* 2009;102: 427-444.
28. Wynn, K. Children's understanding of counting. *Cognition* 1990;36:155-193.
29. Klahr D, Wallace JG. *Cognitive development: An information processing approach*. Hillsdale, NJ: Erlbaum; 1976.

30. Mix KS, Sandhofer CM, Moore JA. How input helps and hinders acquisition of the cardinal word principle. Paper presented at: The biennial meeting of the Society for Research in Child Development. April 2-4, 2009. Denver, CO.
31. Schaeffer B, Eggleston VH, Scott JL. Number development in young children. *Cognitive Psychology* 1974;6:357-379.
32. Spelke ES, Tsivkin S. Initial knowledge and conceptual change: Space and Number. In: Bowerman M, Levinson SC, eds. *Language acquisition and conceptual development.*, New York, NY: Cambridge University Press; 2001:70-97.
33. Wagner S, Walters JA. A longitudinal analysis of early number concepts: From numbers to number. In: Forman G, ed. *Action and thought*. New York: Academic Press; 1982:137-161.
34. LeCorre M, Carey S. One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition* 2007;105: 395-438.
35. Siegel LS. The sequence of development of certain number concepts in preschool children. *Developmental Psychology* 1971;5:357-361.
36. Jordan NC, Kaplan D, Ramineni C, Locuniak MN. Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology* 2009;45: 850-867.
37. Clearfield MW, Mix KS. Number versus contour length in infants' discrimination of small visual sets. *Psychological Science* 1999;10:408-411.
38. Feigenson L, Carey S, Hauser M. The representations underlying infants' choice of more: Object files versus analog magnitudes. *Psychological Science* 2002;13:150-156.
39. Mix KS, Huttenlocher J, Levine SC. Multiple cues for quantification in infancy: Is number one of them? *Psychological Bulletin* 2002;128: 278-294.
40. Barner D, Libenson A, Cheung P, Takasaki M. Cross-linguistic relations between quantifiers and numerals in language acquisition: Evidence from Japanese. *Journal of Experimental Child Psychology* 2009;103: 421-440.

Learning Trajectories in Early Mathematics - Sequences of Acquisition and Teaching

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Introduction

Children follow natural developmental progressions in learning and development. For example, children first learn to crawl, then walk, run, skip, and jump with increasing speed and dexterity. Similarly, they follow natural developmental progressions in learning math; they learn mathematical ideas and skills in their own way.¹ When educators understand these developmental progressions, and sequence activities based on them, they can build mathematically enriched learning environments that are developmentally appropriate and effective. These developmental paths are a main component of a *learning trajectory*.

Key Research Questions

Learning trajectories help us answer several questions.

1. What goals should we establish?
2. Where do we start, that is, what are children's developmental level?
3. How do we know where to go next?
4. How do we get there?

Recent Research Results

Recently, researchers have come to a basic agreement on the nature of learning trajectories.² Learning trajectories have three parts: a) a mathematical goal; b) a developmental path along which children develop to reach that goal; and c) a set of instructional activities, matched to each of the levels of thinking in that path, that help children develop higher levels of thinking. Let's examine each of these three parts.

Goals: The Big Ideas of Mathematics

The first part of a learning trajectory is a *mathematical goal*. Goals involve the *big ideas of mathematics*—clusters of concepts and skills that are mathematically central and coherent, consistent with children’s thinking, and generative of future learning. These big ideas come from several national efforts.³⁻⁶ For example, one big idea is that *counting can be used to find out how many are in a collection. Another would be, geometric shapes can be described, analyzed, transformed and composed and decomposed into other shapes*. It is important to realize that there are several such big ideas and learning trajectories.

Development Progressions: The Paths of Learning

The second part of a learning trajectory consists of levels of thinking; each more sophisticated than the last, through which most children progress on their way to achieving the mathematical goal. That is, the developmental progression describes a typical path children follow in developing understanding and skill about that mathematical topic. Development of mathematics abilities begins when life begins. Young children have certain mathematical-like competencies in number, spatial sense, and patterns from birth.^{1,4}

However, young children's ideas and their interpretations of situations are uniquely different from those of adults. For this reason, early childhood teachers are careful not to assume that children “see” situations, problems, or solutions as they do. Instead, teachers interpret what the child is doing and thinking, they attempt to see the situation from the child’s point of view. Similarly, when these teachers interact with the child, they also consider the instructional activities and their own actions through the child’s eyes. This makes early childhood teaching both demanding and rewarding.

Learning trajectories provide simple labels and descriptions for each level of thinking in every mathematical topic. Figure 1 illustrates a part of the learning trajectory for counting. The Developmental Progression column provides both a label and description for each level. It is important to note that the ages in the first column are approximate. Without experience, some children can be years behind this average age. With high-quality education, children can far exceed these averages. [For complete learning trajectories for all topics, including the research on which they are based, see references^{1,7}, as well as [LearningTrajectories.org](https://www.learningtrajectories.org)].

Instructional Activities: The Paths of Teaching

The third part of a learning trajectory consists of set of instructional strategies and activities, matched to each of the levels of thinking in the developmental progression, designed to help children learn the ideas and skills needed to achieve that level of thinking. That is, as teachers, we can use these strategies and activities to promote children's growth from one level to the next. The third column in Figure 1 provides examples.

Table 1. Sample Levels from the Learning Trajectory for Counting [From references^{1,7} as well as LearningTrajectory.org]

Age	Developmental Progression	Instructional Activities
1 year	<p>Number Word Sayer: Foundations. No verbal counting but names some number words.</p> <p>Chanter Chants in singsong fashion or sometimes-indistinguishable number words.</p>	<p>Associate number words with small quantities (see “Subitizing” in the resources) and verbally count for fun (e.g., going up stairs).</p>
2	<p>Reciter Verbally counts with separate words, not necessarily in the correct order.</p>	<p>Provide repeated, frequent experience with the counting sequence in varied contexts.</p> <p><i>Count and Race.</i> Children verbally count along with the computer (up to 50) by adding cars to a racetrack one at a time.</p>
3	<p>Reciter (10) Verbally counts to ten, with <i>some</i> correspondence with objects.</p>	<p><i>Count and Move.</i> Have all children count from 1-10 or an appropriate number, making motions with each count. For example, say, “one” [touch head], “two” [touch shoulders], “three” [touch head], and so forth.</p>

Age	Developmental Progression	Instructional Activities
	<p>Corresponder Keeps one-to-one correspondence between counting words and objects (one word for each object), at least for small groups of objects laid in a line.</p>	<p><i>Counting Wand.</i> Children use a counting wand to count the number of children in a group, focusing on the 1-to-1 correspondence.</p>
4	<p>Counter (Small Numbers) Accurately counts objects in a line to 5 and answers the “how many” question with the last number counted.</p>	<p><i>Cubes in the Box.</i> Have the child count a small set of cubes. Put them in the box and close the lid. Then ask the child how many cubes you are hiding. If the child is ready, have him/her write the numeral. Dump them out and count together to check.</p> <p><i>Pizza Pizzazz 2</i> Children count items up to 5, putting toppings on a pizza to match a target amount.</p>
	<p>Producer –Counter To (Small Numbers) Counts out objects to 5. Recognizes that counting is relevant to situations in which a certain number must be placed.</p>	<p><i>Count Motions.</i> While waiting during transitions, have children count how many times you jump or clap, or some other motion. Then have them do those motions the same number of times. Initially, count the actions with children.</p>

Age	Developmental Progression	Instructional Activities
5	<p>Counter and Producer (10+) Counts and counts out objects accurately to 10, then beyond (to about 30). Has explicit understanding of cardinality (how numbers tell how many).</p> <p>Keeps track of objects that have and have not been counted, even in different arrangements.</p>	<p><i>Counting Towers (Beyond 10).</i> To allow children to count to 20 and beyond, have them make towers with other objects such as coins. Children build a tower as high as they can, placing more coins, but not straightening coins already in the tower. The goal is to estimate and then count to find out how many coins are in your tallest tower.</p>

In summary, learning trajectories describe the goals of learning, the thinking and learning processes of children at various levels, and the learning activities in which they might engage. People often have several questions about learning trajectories.

How Do Learning Trajectories' Developmental Levels Support Teaching and Learning? The levels help teacher understand children's thinking; create, modify, or sequence activities. Teachers who understand learning trajectories are more effective and efficient and engage children in mathematics joyfully. Through planned teaching and also by encouraging informal, incidental math, teachers help children learn at an appropriate and deep level.

There are Ages in the Learning Trajectories. Should I Plan to Help Children Develop Just the Levels that Correspond to my Children's Ages? The ages in the table are typical ages at which children develop these ideas. But these are rough guides only—children differ widely. Furthermore, the children achieve much later levels with high-quality education. So, these are approximate levels to help orient educators not goals. Children who are provided high-quality math experiences are capable of developing to levels one or more years beyond their peers.

Are the Instructional Activities the Only Way to Teach Children to Achieve Higher Levels of Thinking? No, there are many ways. In some cases, however, there is some research evidence that these are especially effective ways. In other cases, they are simply illustrations of the kind of activity that would be appropriate to reach that level of thinking. Further, teachers need to use a variety of pedagogical strategies in teaching the content, presenting the activities, guiding

children in completing them, and so forth.

Future Directions

Although learning trajectories have proven to be effective for early mathematics curricula and professional development⁸⁻¹⁰, much remains to be studied, such as learning trajectories for older students. Also, in the early years, several learning trajectories are based on considerable research, such as those for counting and arithmetic. However, others, such as patterning have a smaller research base. These remain challenges to the field.

Conclusions

Learning trajectories hold promise for improving professional development and teaching in the area of early mathematics.^{8,11,12} Further, researchers suggest that professional development focused on learning trajectories increases not only teachers' professional knowledge but also their students' motivation and achievement.^{9,13-15} Thus, learning trajectories can facilitate engaging and developmentally appropriate teaching and learning for all children.

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References

1. Clements DH, Sarama J. *Learning and teaching early math: The learning trajectories approach*. 3rd ed. New York, NY: Routledge; 2020.
2. Maloney AP, Confrey J, Nguyen KH, eds. *Learning over time: Learning trajectories in mathematics education*. New York, NY: Information Age Publishing; 2014.
3. Clements DH, Sarama J, DiBiase A-M. *Engaging young children in mathematics: Standards for early childhood mathematics education*. Mahwah, NJ: Lawrence Erlbaum Associates Publishers; 2004.

4. Cross CT, Woods TA, Schweingruber H. *Mathematics learning in early childhood: Paths toward excellence and equity*. Washington DC: National Research Council of the National Academics; 2009.
5. NCTM. *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics; 2000.
6. NGA/CCSSO. *Common core state standards*. Washington, DC: National Governors Association Center for Best Practices, Council of Chief State School Officers; 2010.
7. Sarama J, Clements DH. *Early childhood mathematics education research: Learning trajectories for young children*. New York, NY: Routledge; 2009.
8. Clements DH, Sarama J, Layzer C, Unlu F. Implementation of a scale-up model in early childhood: Long-term impacts on mathematics achievement. *Journal for Research in Mathematics Education*. 2023;54(1):64-88.
9. Sarama J, Clements DH, Guss SS. Longitudinal evaluation of a scale-up model for professional development in early mathematics. In: Dunekacke S, Jegodtka A, Koinzer T, Eilerts K, Jenßen L, eds. *Early childhood teachers' professional competence in mathematics*. London, England: Routledge: 2022:163-186.
10. Dumas DG, McNeish D, Sarama J, Clements DH. Preschool mathematics intervention can significantly improve student learning trajectories through elementary school. *AERA Open*. 2019;5(4):1-5.
11. Clements DH, Sarama J, Baroody AJ, Joswick C. Efficacy of a learning trajectory approach compared to a teach-to-target approach for addition and subtraction. *ZDM Mathematics Education*. 2020;52(4):637-648.
12. Verschaffel L, Bojorquea G, Torbeyns J, Van Hoof J. Persistence of the Building Blocks' impact on Ecuadorian children's early numerical abilities. In: *Proceedings of the EARLI 2019*. Aachen University, Germany:10-16.

13. Clarke BA. A shape is not defined by its shape: Developing young children's geometric understanding. *Journal of Australian Research in Early Childhood Education*. 2004;11(2):110-127.
14. Fennema EH, et al. A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*. 1996;27:403-434.
15. Wright RJ, Martland J, Stafford AK, Stanger G. *Teaching number: Advancing children's skills and strategies*. London: Sage Publications; 2002.

Fostering Early Numeracy in Preschool and Kindergarten

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Introduction

How to best help students learn the single-digit (basic) addition facts, such as $3+4=7$ and $9+5=14$, and related subtraction facts, such as $7-3=4$ and $14-9=5$, has long been debated (see, e.g., Baroody & Dowker,¹ particularly chapters 2, 3, 6, and 7). Nevertheless, there is general agreement that children need to achieve fact fluency.² Fact fluency entails generating sums and differences efficiently (quickly and accurately) and applying this knowledge appropriately and flexibly. Over the last four decades, it has become increasingly clear that children's everyday (informal) mathematical knowledge is an important basis for learning school (formal) mathematics.^{3,4,5} For example, research indicates that helping children build number sense can promote fact fluency.^{6,7,8,9} The aim of this entry is to summarize how the development of informal number sense before grade 1 provides a foundation for the key formal skill of fact fluency in the primary grades.

Key Research Questions

1. When should parents and early childhood educators begin (a) the process of promoting number sense and (b) efforts to foster fact fluency directly?
2. What are the developmental prerequisites preschoolers and kindergartners need to learn in order to efficiently and effectively achieve fact fluency?
3. What role does language play in the development of this foundational knowledge?
4. How can parents and early childhood educators most effectively encourage number sense and fact fluency?

Recent Research Results

Question 1. The process of helping children build number sense—the foundation of fact fluency—can and should begin in the preschool years. Recent research indicates that children begin to construct number sense very early. Indeed, some toddlers as young as 18 months and nearly all 2-year-olds have begun learning the developmental prerequisites for fact fluency (e.g., see Baroody, Lai, & Mix,³ for a review).

Successful efforts to promote fact fluency depend on ensuring a child is developmentally ready and on not rushing the child. As research indicates significant individual differences in number sense appear as early as two or three years of age and often increase with age,^{3,10} there are no hard and fast rules about when formal training on fact fluency should begin. For many children though, this training, even with the easiest ($n+0$ and $n+1$) sums, may not be developmentally appropriate until late kindergarten or early first grade.¹¹ For children at risk for academic failure, work with even the easiest sums often does not make sense until first or second grade.¹²

Questions 2 and 3. Some research indicates that language, in the form of the first few number words, plays a key role in the construction of number sense (for a detailed discussion, see Baroody,³ Mix, Sandhofer, & Baroody¹³). More specifically, it can provide a basis for two foundations of early number sense—namely a concept of cardinal number (the total number of items in a collection) and the skill of verbal number recognition (VNR), sometimes called “(verbal) subitizing,” shown at the apex of Figure 1. VNR entails reliably and efficiently recognizing the number of items in small collections and labeling them with the appropriate number word. The use of “one,” “two,” “three” in conjunction with seeing examples and non-examples of each can help 2- and 3-year-olds construct an increasingly reliable and accurate concept of the “intuitive numbers” *one*, *two*, and *three*—an understanding of oneness, twoness, and threeness.

- By seeing $\bullet\bullet$, $\Delta\Delta$, and $\bigcirc\bigcirc$ (examples of pairs), for instance, all labeled “two,” young children can recognize that the appearance of the items in the collections is not important (shape and color are irrelevant to number). It can also provide them a label (“two”) for their intuitive idea of *plurality* (more than one item).
- Seeing \bullet , $\bullet\bullet\bullet$, Δ , $\Delta\Delta\Delta$, $\square\square$, and \square (non-examples of pairs) labeled as “not two” or with another number word can help them define the boundaries of the concept of *two*.

The key instructional implications are that a basic understanding of cardinal number is not innate nor does it unfold automatically (cf. Dehaene¹⁵).^{14,16} Parents and preschool teachers are important

in providing the experiences and feedback needed to construct number concepts. They should take advantage of meaningful everyday situations to label (and encourage children) to label small collections (e.g., “How many feet do you have?” “So you need two shoes, not one.” “You may take one cookie, not two cookies.”). Some children enter kindergarten without being able to recognize all the intuitive numbers. Such children are seriously at risk for school failure and need intensive remedial work. Kindergarten screening should check for whether children can immediately recognize collections of one to three items and be able to distinguish them from somewhat larger collections of four or five.

As **Figure 1** illustrates, the co-evolution of cardinal concepts of the intuitive numbers and the skill of VNR can provide a basis for a wide variety of number, counting, and arithmetic concepts and skills. These skills can provide a basis for meaningful verbal counting. Recognition of the intuitive numbers can help children literally see that a collection labeled “two” has more items than a collection labeled “one” and that a collection labeled “three” has more items than a collection labeled “two.” This basic ordinal understanding of number, in turn, can help children recognize that the order of number words matters when we count (the *stable order principle*) and that the number word sequence (“one, two, three...”) represents increasingly larger collections. As a child becomes familiar with the counting sequence, they develop the ability to start at any point in the counting sequence and (efficiently) state the next number word in the sequence (number-after skill) instead of counting from “one.”

The ability to automatically cite the number after another number in counting sequence, can be the basis for the insight that adding “one” to a number results in a larger number and, more specifically, the number-after rule for $n+1/1+n$ facts. When adding “one”, the sum is the number after the other number in the counting sequence (e.g., the sum of $7+1$ is the number after “seven” when we count or “eight”). This reasoning strategy can enable children to efficiently deduce the sum of any such combination for which they know the counting sequence, even those not previously practiced including large multi-digit facts such as $28+1$, $128+1$, or $1,000,128+1$. In time, this reasoning strategy becomes automatic—can be applied efficiently, without deliberation (i.e., becomes a component in the retrieval network). In other words, it becomes the basis for fact fluency with the $n+1/1+n$ combinations.

VNR, and the cardinal concept of number it embodies, can be a basis for meaningful object counting.¹⁷ Children who can recognize “one,” “two,” and “three” are more likely to benefit from adult efforts to model and teach object counting than those who cannot. They are also more likely

to recognize the purpose of object counting (as another way of determining a collection's total) and the rationale for object-counting procedures (e.g., the reason why others emphasize or repeat the last number word used in the counting process is because it represents the collection's total). Meaningful object counting is necessary for the invention of counting strategies (with objects or number words) to determine sums and differences. As these strategies become efficient, attention is freed to discover patterns and relations; these mathematical regularities, in turn, can serve as the basis for reasoning strategies (i.e., using known facts and relations to deduce the answer of an unknown combination). As these reasoning strategies become automatic, they can serve as one of the retrieval strategies for efficiently producing answers from a memory or retrieval network.

VNR can enable a child to see one & one as *two*, one & one & one as *three*, or two & one as *three* and the reverse (e.g., *three* as one & one & one or as two & one). The child thus constructs an understanding of composition and decomposition (a whole can be built up from, or broken down into, individual parts, often in different ways). Repeatedly seeing the composition and decomposition of *two* and *three* can lead to fact fluency with the simplest addition and subtraction facts (e.g., "one and one is two," "two and one is three," and "two take away one is one"). Repeatedly decomposing *four* and *five* with feedback (e.g., labeling a collection of four as "two and two", and hearing another person confirm, "Yes, two and two makes four") can lead to fact fluency with the simplest sums to five and is one way of discovering the number-after rule for $n+1/1+n$ combinations (discussed earlier).

The concept of cardinality, VNR, and the concepts of composition and decomposition can together provide the basis for constructing a basic concept of addition and subtraction. For example, by adding an item to a collection of two items, a child can literally see that the original collection has been transformed into a larger collection of *three*. These competencies can also provide a basis for constructing a relatively concrete, and even a relatively abstract, understanding of the following arithmetic concepts¹⁸:

- Concept of subtractive negation. For example, when children recognize that if you have two blocks and take away two blocks this leaves none, they may induce the pattern that *any number take away itself leaves nothing*.
- Concept of additive and subtractive identity. For example, when children recognize that two blocks take away none leaves two blocks, they may induce the regularity that if none is taken from any number, *the number will remain unchanged*. The concepts of subtractive

negation and subtractive identity can provide a basis for fact fluency with the $n - n = 0$ and $n - 0 = n$ families of subtraction facts, respectively.

A weak number sense, then, can interfere with achieving fact fluency and other aspects of mathematical achievement. For example, Mazzocco and Thompson¹⁹ found that preschoolers' performance on the following four items of the Test of Early Mathematics Ability—Second Edition (TEMA-2) was predictive of which children would have mathematical difficulties in second and third grade: meaningful object counting (recognizing that the last number word used in the counting process indicates the total), cardinality, comparing of one-digit numbers (e.g., Which is more five or four?), mentally adding one-digit numbers, and reading one-digit one numerals. Note that verbal number recognition of the intuitive numbers is a foundation for the first three skills and meaningful learning of the fourth.

Question 4. The basis for helping students build both number sense in general and fact fluency in particular is creating opportunities for them to discover patterns and relations. For example, a child who has learned the “doubles,” such as $5+5=10$ and $6+6=12$, in a meaningful manner (e.g., the child recognizes that the sums of this family are all even or count-by-two numbers) can use this knowledge to reason out the sums of unknown doubles-plus-one facts, such as $5+6$ or $7+6$.

In order to be developmentally appropriate, such learning opportunities should be purposeful, meaningful, and inquiry-based.²⁰

- Instruction should be purposeful and engaging to children. This can be achieved by embedding instruction in structured play (e.g., playing a game that involves throwing a die can help children learn to recognize regular patterns of one to six). Music and art lessons can serve as natural vehicles for thinking about patterns, numbers, and shapes (e.g., keeping a beat of two or three; drawing groups of balloons). Parents and teachers can take advantage of numerous everyday situations (e.g., “How many feet do you have? ...So, how many socks should you get from your sock drawer?”). Children’s questions can be an important source of purposeful instruction.
- Instruction should be meaningful to children, building gradually on (and being connected to) what they know. A meaningful goal for adults working with 2-year-olds is to have children recognize two. Pushing them too fast to recognize larger numbers such as *four* can be overwhelming, causing them to melt down (become inattentive or aggressive, guess wildly, or otherwise disengage from the activity).

- Instruction should be inquiry based, or thought provoking, to the extent possible. Instead of simply feeding children information, parents and teachers should give children an opportunity to think about a problem or task, make conjectures (educated guesses), devise their own strategy or deduce their own answer.

The various points above are illustrated by the case of Alice²¹ and Lukas.²²

- *The case of Alice.* The 2.5-year-old had for several months been able to recognize one, two, or three things. So her parents wanted to expand her number range to four, which was now just outside her range of competence. Instead of simply labeling collections of four for her, they asked her about collections of four. Alice often responded by decomposing the unrecognizable collections into two familiar collections of two. Her parents then built on her response by saying, “Two and two is four.” At 30 months of age, *shown a picture of four puppies, Alice put two fingers of her left hand on two dogs and said, “Two.” While maintaining this posture, she placed two fingers of her right hand on the other two puppies and said, “Two.” She then used the known relation “2 and 2 makes 4” (learned from her parents) to specify the cardinal value of the collection.*
- *The case of Lukas.* In the context of a computer-based math game, Lukas was presented $6+6$. He determined the sum by counting. Shortly afterward, he was presented $7+7$. He smiled and answered quickly, “Thirteen.” When the computer feedback indicated the sum was 14, he seemed puzzled. A couple of items later, he was presented $8+8$ and noted, “I was going to say 15, because $7+7$ was 14. But before $6+6$ was 12, I thought for sure that $7+7$ would be 13 but it was 14. So I’m going to say $8+8$ is 16.”

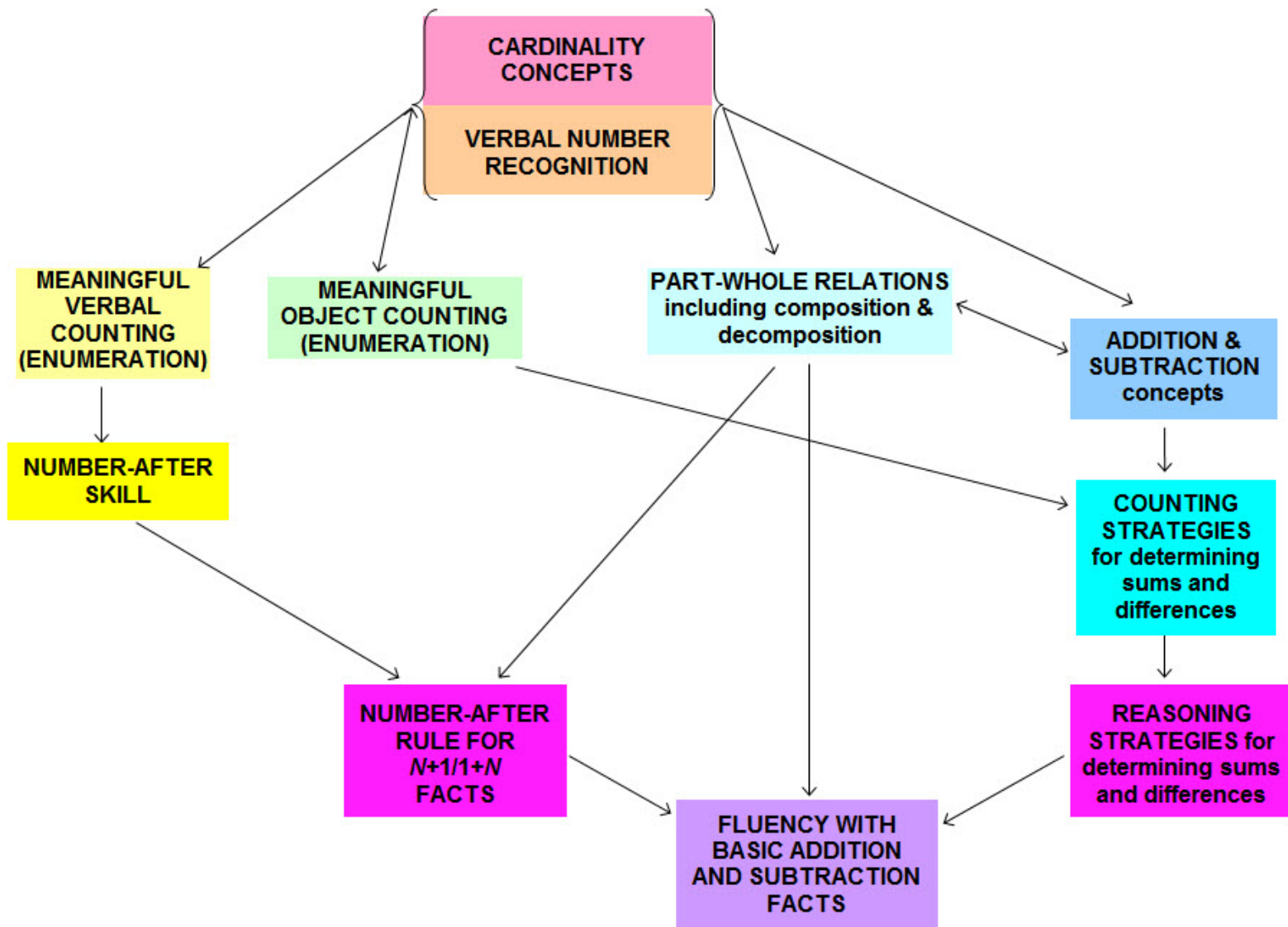
Future Directions

Much still needs to be learned about preschoolers’ mathematical development. Does VNR ability at two years of age predict readiness for kindergarten or mathematical achievement in school? If so, can intervention that focuses on examples and non-examples enable children at risk for academic failure to catch up with their peers? What other concepts or skills at two or three years of age might be predictive of readiness for kindergarten or mathematical achievement in school? How effective are the early childhood mathematics programs currently being developed?

Conclusions

Contrary to the beliefs of many early childhood educators, mathematics instruction for children as young as two years of age does make sense.^{23,24,25,26} As **Figure 1** makes clear, this instruction should start with helping children construct a cardinal concept of the intuitive numbers and the skill of recognizing and labeling sets of one to three items with an appropriate number word. As Figure 1 further illustrates, these aspects of number knowledge are key to later numeracy and often lacking among children with mathematical disabilities.²⁷ Early instruction does not mean imposing knowledge on preschoolers, drilling them with flashcards, or otherwise having them memorize by rote arithmetic facts. Fostering number sense and fact fluency both should focus on helping children discover patterns and relations and encouraging their invention of reasoning strategies.

Figure 1. Learning Trajectory of Some Key Number, Counting, & Arithmetic Concepts and Skills



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References

1. Baroody AJ, Dowker A. The development of arithmetic concepts and skills: Constructing adaptive expertise. In: Schoenfeld A, ed. *Studies in mathematics thinking and learning series*. Mahwah, NJ: Lawrence Erlbaum Associates; 2003.
2. Kilpatrick J, Swafford J, Findell B, eds. *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press; 2001.

3. Baroody AJ, Lai ML, Mix KS. The development of number and operation sense in early childhood. In: Saracho O, Spodek B, eds. *Handbook of research on the education of young children*. Mahwah, NJ: Erlbaum; 2006: 187-221.
4. Clements D, Sarama J, DiBiase AM, eds. *Engaging young children in mathematics: Standards for early childhood mathematics education*. Mahwah, NJ: Lawrence Erlbaum Associates; 2004: 149-172.
5. Ginsburg HP, Klein A, Starkey P. The development of children's mathematical knowledge: Connecting research with practice. In: Sigel IE, Renninger KA, eds. *Child psychology in practice*. 5th Ed. New York, NY: Wiley & Sons; 1998; 401-476. *Handbook of child psychology*, vol 4.
6. Baroody AJ. Why children have difficulties mastering the basic number facts and how to help them. *Teaching Children Mathematics* 2006;13:22-31.
7. Baroody AJ, Thompson B, Eiland M. Fostering the fact fluency of grade 1 at-risk children. Paper presented at: The annual meeting of the American Educational Research Association. April, 2008. New York, NY.
8. Gersten R, Chard, D. Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *The Journal of Special Education* 1999;33(1):18-28.
9. Jordan NC. The need for number sense. *Educational Leadership* 2007;65(2):63-66.
10. Dowker AD. *Individual differences in arithmetic: Implications for psychology, neuroscience and education*. Hove, England: Psychology Press; 2005.
11. Baroody AJ. The development of kindergartners' mental-addition strategies. *Learning and Individual Differences* 1992;4:215-235.
12. Baroody AJ, Eiland M, Thompson B. Fostering at-risk preschoolers' number sense. *Early Education and Development* 2009;20:80-120.
13. Mix KS, Sandhofer CM, Baroody AJ. Number words and number concepts: The interplay of verbal and nonverbal processes in early quantitative development. In: Kail R, ed. *Advances in child development and behavior*, vol 33. New York, NY: Academic Press; 2005: 305-346.
14. Baroody AJ, Li X, Lai ML. Toddlers' spontaneous attention to number. *Mathematics Thinking and Learning* 2008;10:1-31.
15. Dehaene S. *The number sense*. New York, NY: Oxford University Press; 1997.
16. Wynn K. Numerical competence in infants. In; Donlan C, ed. *Development of mathematical skills*. Hove, England: Psychology Press; 1998: 1-25.
17. Benoit L, Lehalle H, Jouen F. Do young children acquire number words through subitizing or counting? *Cognitive Development* 2004;19:291-307.
18. Baroody AJ, Lai ML, Li X, Baroody AE. Preschoolers' understanding of subtraction-related principles. *Mathematics Thinking and Learning* 2009;11:41-60.
19. Mazzocco M, Thompson R. Kindergarten predictors of math learning disability. *Learning Disabilities Research & Practice* 2005;20:142-155.
20. Baroody AJ, Coslick RT. *Fostering children's mathematical power: An investigative approach to K-8 mathematics instruction*. Mahwah, NJ: Erlbaum; 1998.
21. Baroody AJ, Rosu L. Adaptive expertise with basic addition and subtraction combinations: The number sense view. In: Baroody AJ, Torbeyns T. chairs. *Developing Adaptive Expertise in Elementary School Arithmetic*. Symposium conducted at: The annual meeting of the American Educational Research Association. April, 2006. San Francisco, CA.
22. Baroody AJ. Fostering early number sense. Keynote address at: The Banff International Conference on Behavioural Science. March, 2008. Banff, Alberta.

23. Baroody AJ, Li X. Mathematics instruction that makes sense for 2 to 5 year olds. In: Essa EA, Burnham MM, eds. *Development and education: Research reviews from young children*. New York: The National Association for the Education of Young Children; 2009: 119-135.
24. Bredekamp S, Copple C. *Developmentally appropriate practice in early childhood programs*. Washington, DC: National Association for the Education of Young Children; 1997.
25. Copley J, ed. *Mathematics in the early years, birth to five*. Reston, VA: National Council of Teachers of Mathematics; 1999.
26. Copley J, ed. *The young child and mathematics*. Washington, DC: National Association for the Education of Young Children; 2000.
27. Landerl K, Bevan A, Butterworth B. Developmental dyscalculia and basic numerical capacities: A study of 8-9 year old students. *Cognition* 2004;93:99-125.

Mathematics Instruction for Preschoolers

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Introduction

Teaching mathematics to young children, prior to formal school entry, is not a new practice. In fact, early childhood mathematics education (ECME) has been around in various forms for hundreds of years.¹ What has altered over time are opinions related to why ECME is important, what mathematics education should accomplish, and how (or whether) mathematics instruction should be provided for such young audiences.

Subject and Research Context

Is ECME necessary?

A concern among many early childhood experts, including educators and researchers, is the recent trend toward the “downward extension of schooling”² such that curricula, and the corresponding focus on assessment scores that were formally reserved for school-aged children, are now being pushed to preschool levels.³ The motivation behind this downward push of curriculum appears to be largely political, with an increasing emphasis on early success, improving test scores, and closing gaps among specific minority and socio-economic groups.⁴

Despite the concern related to the downward extension of school-aged curricula in general, there are persuasive factors encouraging the presence of at least some type of mathematical instruction for preschoolers, or at least for some groups of preschoolers. As Ginsburg et al. point out, learning mathematics is “a ‘natural’ and developmentally appropriate activity for young children”,¹ and through their everyday interactions with the world, many children develop informal concepts about space, quantity, size, patterns, and operations. Unfortunately, not all children have the same opportunities to build these informal, yet foundational, concepts of mathematics in their day-to-day lives. Subsequently, and because equity is such an important aspect of mathematics education, ECME seems particularly important for children from marginalized groups,³ such as special needs children, English-as-additional-language (EAL) learners, and

children from low socio-economic status (SES), unstable, or neglectful homes.⁴

Recent Research Results

Equity in education is one major argument for the presence of ECME, but intimately tied to equity is the aspect of helping young mathematical minds move from informal to formal concepts of mathematics, concepts that have names, principles and rules. Children’s developing mathematical concepts, often building on informal experiences, can be represented as learning trajectories⁵ that highlight how specific mathematical skills can build upon preceding experiences and inform subsequent steps. For example, learning the names, order and quantities of the “intuitive numbers” 1-3, and recognizing these values as sets of objects, number words, and as parts of wholes (e.g., three can be made up of 2 and 1 or $1 + 1 + 1$), can help children develop an understanding of simple operations.⁶ “Mathematizing,” or providing appropriate mathematical experiences and enriching those experiences with mathematical vocabulary, can help connect children’s early and naturally occurring curiosities and observations about math to later concepts in school.³ Researchers have found evidence to suggest very early mathematical reasoning,^{1,6,7} and ECME can help children formalize early concepts, make connections among related concepts, and provide the vocabulary and symbol systems necessary for mathematical communication and translation (for an example, see Baroody’s paper⁶).

ECME may be important for reasons beyond equity and mathematization. In an analysis of six longitudinal studies, Duncan et al.⁸ found that children’s school-entry math skills predicated later academic performance more strongly than attentional, socioemotional or reading skills. Similarly, early difficulty with foundation mathematical concepts can have lasting effects as children progress through school. Given that math skills are so important for productive participation in the modern world (Platas L, unpublished data, 2006),⁹ and that specific mathematical domains, such as algebra, can serve as a gatekeeper to higher education and career options,¹⁰ early, equitable and appropriate mathematical experiences for all young children are of critical importance.

What is “appropriate” ECME?

Views differ with respect to what ECME should consist of and how it should be infused into preschoolers’ lives, with a continuum that represents the amount of intervention or instruction proposed. On one end of the continuum is a very direct, didactic, and teacher-centered approach to ECME, while the other end of the spectrum represents a play-based, child-centered, non-

didactic approach to ECME.⁴ Individual children, and perhaps different groups of children, may benefit from varying levels of instruction throughout the continuum, and much research remains to be done to better understand best practices for all children and all aspects of mathematics. One example of a research-based mathematics curriculum for young children is Building Blocks, a program designed to support and enhance children's developing mathematical thinking (i.e., learning trajectories) through the use of computer games, everyday objects (i.e., manipulatives such as blocks), and print.¹¹ Building Blocks represents an attempt to align content and instructional activities with the learning trajectories of well-researched domains such as counting. The learning trajectories of other domains, such as measurement and patterning, are not yet well understood.⁵

Ginsburg et al.¹ described six components that should be present in all forms of ECME (e.g., programs such as Building Blocks), including environment, play, teachable moments, projects, curriculum, and intentional teaching. For example, regardless of where a particular mathematics curriculum falls on the playful-didactic continuum, environment is a vital component of early education. Specifically, providing preschool children with materials that inspire mathematical thinking, such as blocks, shapes, and puzzles, can facilitate the development of foundational skills such as patterning, making comparisons, and early numeracy. Another important component is that of the teachable moment: recognizing and capitalizing on children's spontaneous math-related discoveries by asking questions that require children to reflect and respond, by providing vocabulary and representational support, and by demonstrating extension activities that elaborate on and further support mathematical ideas.

Perhaps the most popular component of ECME in the current literature is play. Many proponents of play-based learning, or learning through play, argue that children learn a great deal when they discover mathematical ideas on their own in natural or minimally contrived situations.^{12,13} Some argue that play is being taken out of preschools in reaction to the downward extension of schooling and testing,¹⁴ and they provide data to suggest that children in early grades (including kindergarten) now spend far more time on test preparation than they do on play-based activities.⁴ Even many educational toys appear marketed more toward early learning of academic concepts (i.e., literacy for toddlers) than toward playful learning per se. This approach may be driven in part by parents' views on the importance of early education for future academic success. Much research remains to be done on the impact of educational toys, technology, play (or lack thereof), and various ECME curricula on preschoolers' mathematical development.

Research Gaps and Implications

What are the barriers to effective early education?

Mathematics for preschool children is complicated by several factors, including political pressure (i.e., achievement scores, funding, varying curriculum standards), individual differences among preschoolers (i.e., individual children may benefit from different mathematical opportunities), ideological differences regarding education (i.e., playful–didactic continuum), and gaps in developmental research (i.e., uncertain learning trajectories for some mathematical concepts). Complicating ECME further are barriers that affect the implementation of mathematical instruction (regardless of curriculum), such as teachers’ own fears or misunderstandings of mathematics. Unfortunately, many preschool educators lack training directly related to mathematics for young children (Platas L, unpublished data, 2006). Teachers need knowledge of what children know, knowledge of how children learn new concepts, knowledge of most effective teaching strategies, and the mathematical concepts themselves (Platas L, unpublished data, 2006).³ Improving the mathematical training opportunities for early educators may help to improve the quality (and quantity) of mathematics instruction for young children.

Conclusion

The debate surrounding ECME does not appear to be about whether early exposure to mathematical experiences and ideas is important; the general consensus is that it is important. Rather, the issue is how, when, why and for whom specific approaches to ECME should be presented. Opinions differ regarding the amount of structure versus free-play and specific curriculum versus teachable moments. Yet as evidence accumulates regarding very young children’s developing mathematical ideas (i.e., learning trajectories), attempts to align cognitive development with best instructional practices (or with the best environments to support natural mathematical discoveries) may help pave the way for equitable and appropriate mathematical experiences for all preschool children.

References

1. Ginsburg HP, Lee JS, Boyd JS. Mathematics education for young children: What it is and how to promote it. *Social Policy Report* 2008;223-23.
2. Elkind D. Foreword. In: Miller E, Almon J, eds. *Crisis in the kindergarten: Why children need to play in school*. College Park, MD: Alliance for Childhood; 2009: 9.
3. Clements DH. Major themes and recommendations. In: Clements DH, Sarama J, DiBiase A, eds. *Engaging young children in mathematics: Standards for early childhood mathematics education*. Mahwah, NJ: Erlbaum; 2004: 7-72.

4. Miller E, Almon J, eds. *Crisis in the kindergarten: Why children need to play in school*. College Park, MD: Alliance for Childhood; 2009:1-72.
5. Clements DH, Sarama J. Learning trajectories in early mathematics – sequences of acquisition and teaching. *Encyclopedia of Language and Literacy Development*. London, ON: Canadian Language and Literacy Research Network; 2009: 1-7.
6. Baroody AJ. Fostering early numeracy in preschool and kindergarten. *Encyclopedia of Language and Literacy Development*. London, ON: Canadian Language and Literacy Research Network; 2009: 1-9.
7. Sophian C. Numerical knowledge in early childhood. In: Tremblay RE, Barr RG, Peters RDeV, Boivin M, eds. *Encyclopedia on Early Childhood Development* [online]. Montreal, Quebec: Centre of Excellence for Early Childhood Development; 2009:1-7.
8. Duncan GJ, Dowsett CJ, Claessens A, Magnuson K, Huston AC, Klebanov P, Pagani LS, Feinstein L, Engel M, Brooks-Gunn J, Sexton H, Duckworth K, Japel C. School readiness and later achievement. *Developmental Psychology* 2007;43:1428-1446.
9. Baroody AJ, Lai M, Mix KS. The development of young children's early number and operation sense and its implications for early childhood education. In: Spodek B, Olivia S, eds. *Handbook of research on the education of young children*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc; 2006:187-221.
10. Knuth EJ, Alibali MW, McNeil NM, Weinberg A, Stephens AC. Middle school students' understanding of core algebraic concepts: Equality and variable. *Zentralblatt für Didaktik der Mathematik* 2005;37:1-9.12.
11. Sarama J. Technology in early childhood mathematics: Building Blocks as an innovative technology-based curriculum. In: Clements DH, Sarama J, DiBiase A, eds. *Engaging young children in mathematics: Standards for early childhood mathematics education*. Mahwah, NJ: Erlbaum; 2004: 361-375.
12. Polonsky L, Freedman D, Leshner S, Morrison K. *Math for the very young: A handbook of activities for parents and teachers*. New York, NY: John Wiley & Sons; 1995.
13. Seo K, Ginsburg HP. What is developmentally appropriate in early childhood mathematics education? Lesson from new research. In: Clements DH, Sarama J, DiBiase A, eds. *Engaging young children in mathematics: Standards for early childhood mathematics education*. Mahwah, NJ: Erlbaum; 2004: 91-104.
14. Hirsh-Pasek K, Golinkoff RM, Berk LE, Singer DG. *A mandate for playful learning in preschool: Presenting the Evidence*. Oxford, UK: University Press; 2009